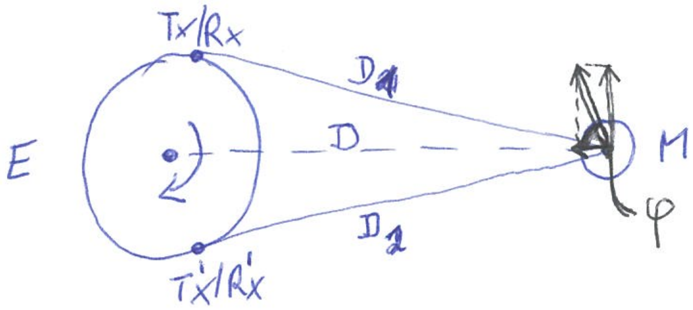
OE2IGL ①
V3 26.2.2024

Moon: Doppler & Libration

Doppler orbit motion shift

due to elliptical path motion of moon around earth



only orbit motion without rotation of moon.

see: engl. WIKI „doppler radar"
or search for „radar tutorial"

echo mode: Tx/Rx

$$\Delta f_{Doppler} = f_C \frac{2v}{c} \cdot \cos\varphi = f_C \frac{2\dot{D}_1}{c}$$

$T'x/R'x$

$$\left(= f_C \cdot \frac{\dot{D}_1 + \dot{D}_2}{2} \right)$$

$$\omega = 2\pi f = 2\pi \frac{1}{24 \cdot 3600 - 4 \cdot 60 + 4}$$

$$V_{max} = \omega R_E = \frac{2\pi \cdot 6378000}{24 \cdot 3600 - 4 \cdot 60 + 4} = 465 \, m/s$$

$$\Delta f_{Doppler} = \frac{465 \cdot 2 \cdot 10368 \cdot 10^6}{3 \cdot 10^8} = \pm 16{,}1 \, kHz \quad @ \; 10{,}368 \, GHz$$

⇒ max Doppler shift
Difference between Tx/Rx echo and T'x/R'x echo

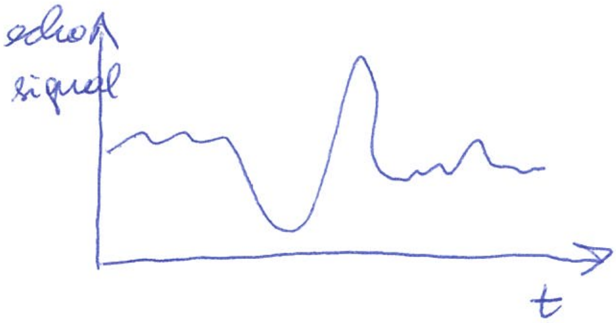
$$\Delta f_{Doppler\,max} = 2 \cdot \Delta f_{Doppler} = \pm 32{,}2 \, kHz$$

Libration

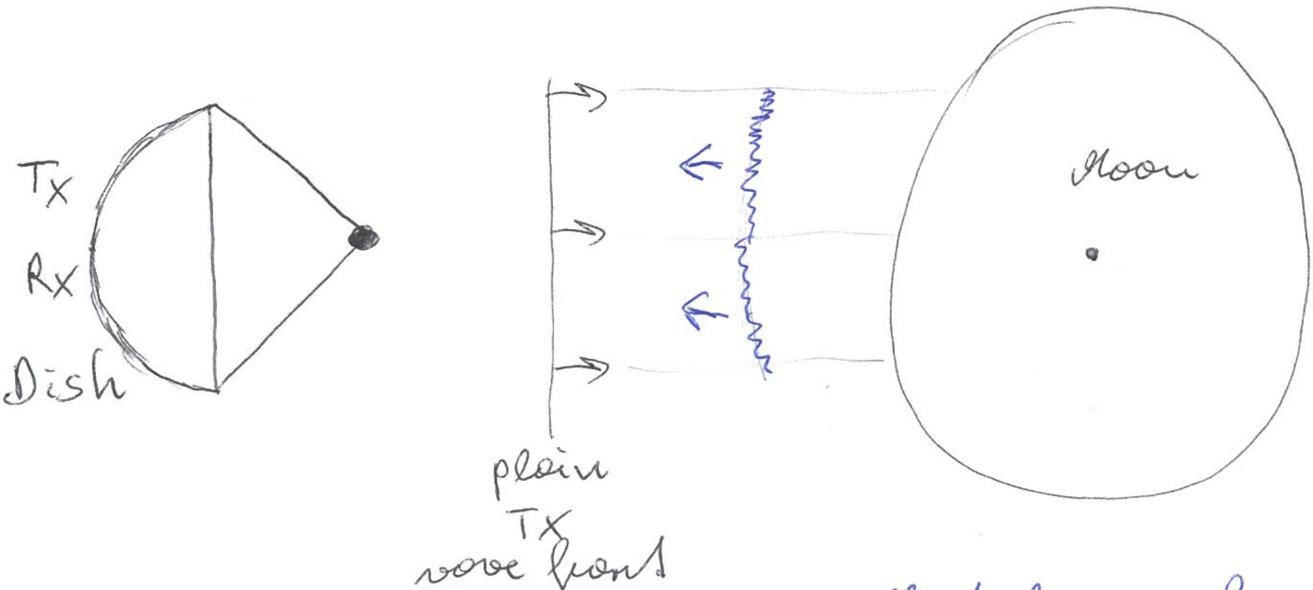
see Wikipedia, Dubus 2018 p. 38, Dubus 3/2010 for details.

1. Libration fading

Wording is misleading, it is more an echo fading:



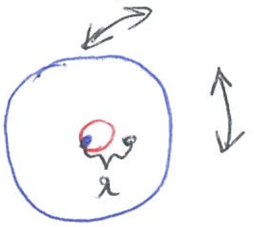
fluctuations over time, no doppler or spreading if v/rotation = 0



reflected wave front with time delay (=phase delay) of curved moon surface + phase delay of unplain moon ground results in interferences in Rx dish focus.

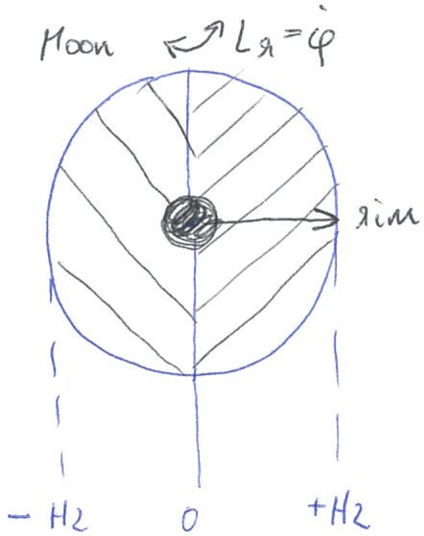
2. Doppler Libration Spreading

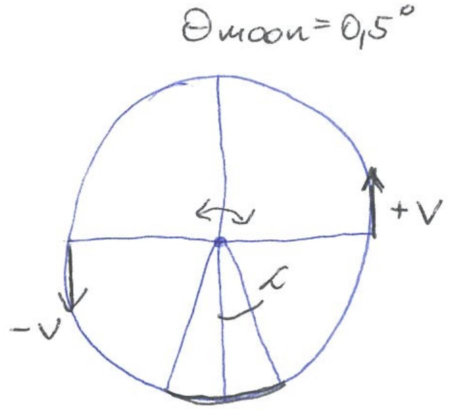
Due to wobbling of the moon there is a spreading of the signal from limb to limb. That means necessary bandwidth is greater than for pure f_c signal.



libration rate $L_R = \dot{\varphi}_{[angle/min]}$ can be calculated

$speed_{(r)} = \dot{\varphi} \cdot r$ ⇒ speed in moon center ≈ 0





Tx and Rx antenna pointing to moon center.

Tx, Rx distance

$$\Delta f_{max} = \text{spreading} = 2 \cdot 2 \cdot f_c \cdot \frac{V}{c} \cdot \sin\alpha = 4 f_c \cdot \frac{L_R[°/min] \cdot \pi \cdot r}{180 \cdot 60 \cdot c}$$

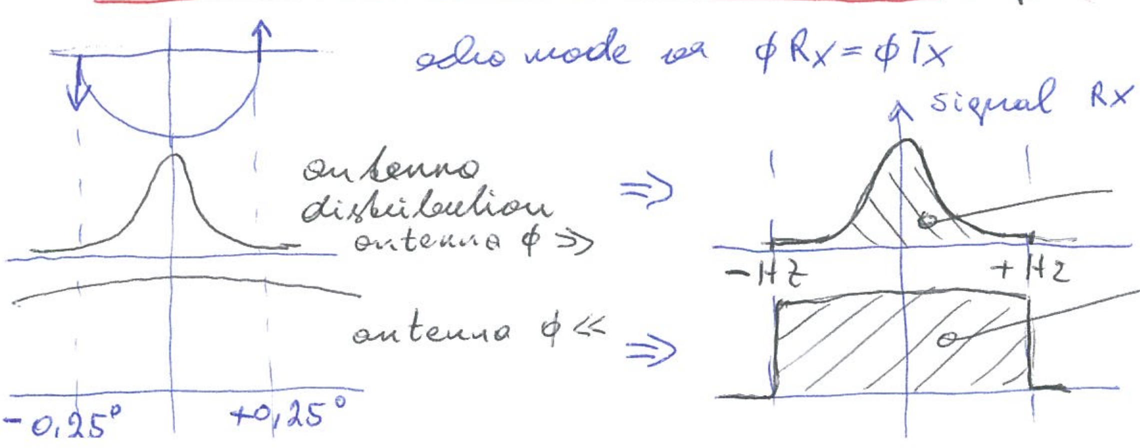
left to right rim

1 for max

$$\boxed{\Delta f_{max\,[Hz]} = 6750 \cdot f_{c\,[GHz]} \cdot L_{R\,[°/min]}}$$

max. Libration Spreading

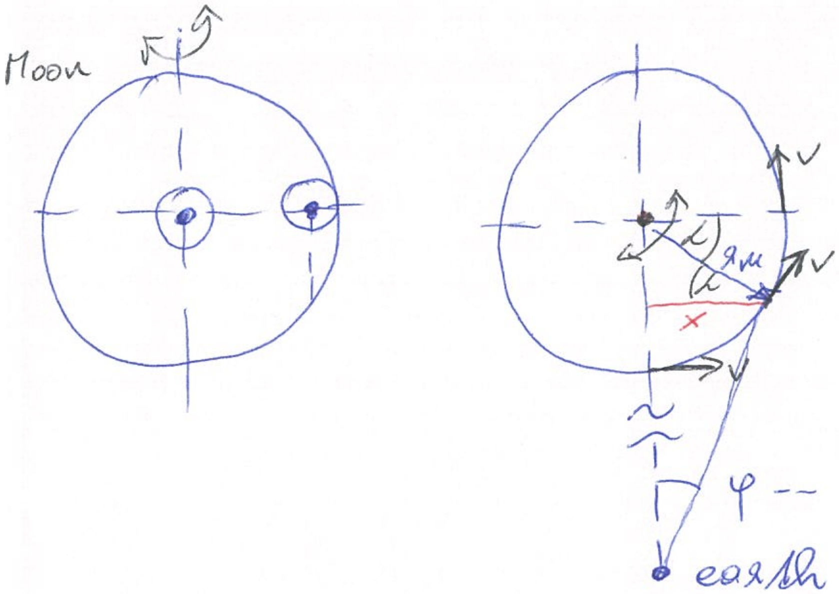
echo mode or $\phi Rx = \phi Tx$



measured power depends on beam width factor BWF.

What happends if Rx and/or Tx is outside of moon center.

Now we have <u>spreading and additional Doppler shift</u>!



$$f_D \sim \frac{v}{c} \cdot \cos \alpha$$

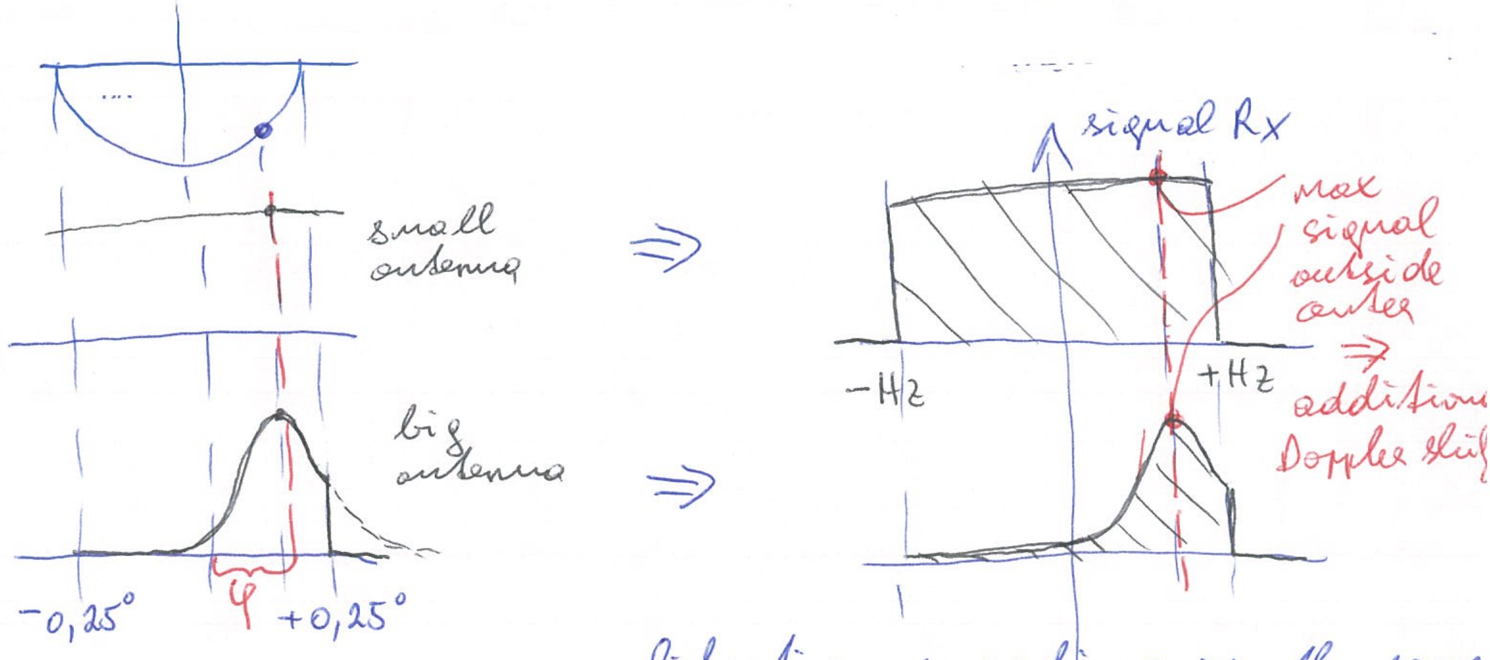
$$\cos \alpha = \frac{x}{r_M} = \frac{x}{D} \cdot \frac{D}{r_M} = \frac{\varphi^\circ}{\theta^\circ_{moon}}$$

φ -- pointing outside of moon center

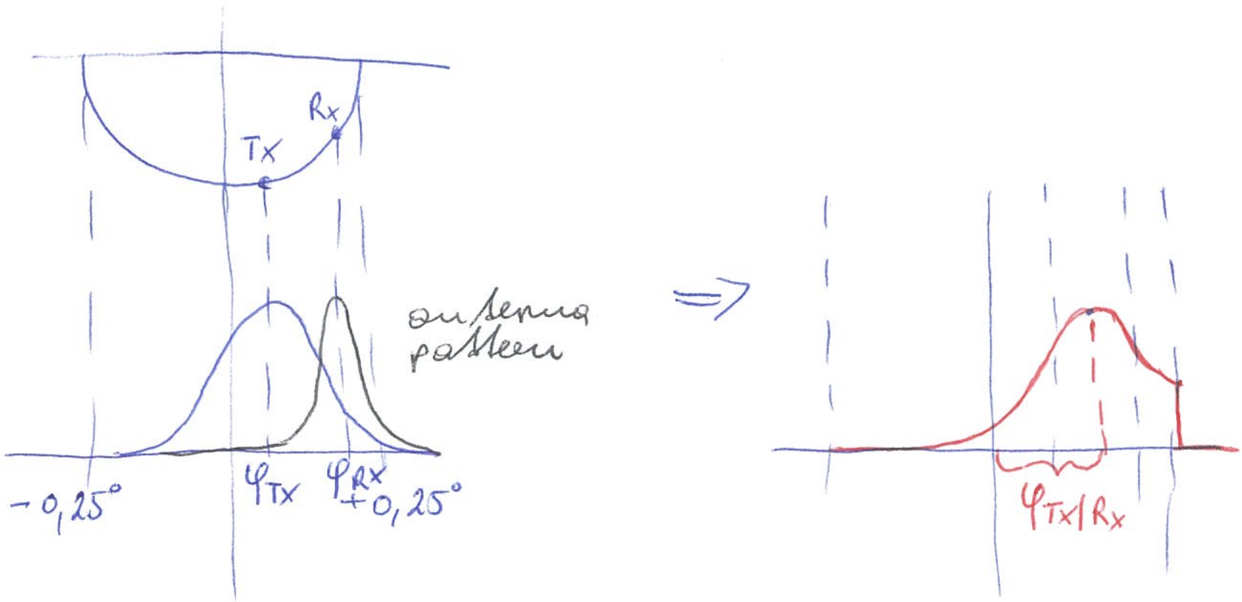
$$\Rightarrow \boxed{\Delta f_{Doppler} = 6750 \cdot f_{c\,[GHz]} \cdot L_{R\,[°/min]} \cdot \frac{\varphi^\circ}{\theta^\circ_{moon}}}$$

<u>echo mode</u>: pointing outside of moon center is equal for Tx and Rx antenna

→ φ – pointing error of antenna.



libration spreading is the same from limb-to-limb (−Hz to +Hz) different Tx and Rx pointing error and antenna diameter:



$\varphi_{Tx|Rx}$ — is the convolution of Tx and Rx antenna pattern and moon reflexion ~~over~~ moon surface

for constant moon reflexion (valid > 30 GHz):

$$T_x(x) = e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} \qquad R_x(x) = e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

$$\frac{\partial\left(T_x(x)\cdot R_x(x)\right)}{\partial x} = 0 \quad \text{to find } \varphi_{Tx|Rx}$$

$$0 = T_x'(x)\cdot R_x(x) + T_x(x)\cdot R_x'(x)$$

$$\Rightarrow x = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}$$

$$\varphi_{Tx|Rx} = \frac{\varphi_{Tx}\cdot HPBW_{Rx}^2 + \varphi_{Rx}\cdot HPBW_{Tx}^2}{HPBW_{Rx}^2 + HPBW_{Tx}^2}$$

$$\boxed{\Delta f_{Doppler} = 6750\cdot f_{c\,[GHz]}\cdot L_{\lambda\,[°/min]}\cdot\frac{\varphi_{Tx}\cdot HPBW_{Rx}^2+\varphi_{Rx}\cdot HPBW_{Tx}^2}{\theta_{moon}\left(HPBW_{Rx}^2+HPBW_{Tx}^2\right)}}$$

-- additional Doppler due to libration